

SM3 2.1: Complex Factoring Techniques

Problems:

Factor each quadratic expression completely.

1) $12m^2 + 12$

$$12(m + i)(m - i)$$

2) $7x^2 + 2$

$$7\left(x - \frac{i\sqrt{14}}{7}\right)\left(x + \frac{i\sqrt{14}}{7}\right)$$

Or

$$(\sqrt{7}x - i\sqrt{2})(\sqrt{7}x + i\sqrt{2})$$

3) $-4x^2 - 8$

$$-4(x - i\sqrt{2})(x + i\sqrt{2})$$

4) $4x^2 + 7$

$$4\left(x - \frac{i\sqrt{7}}{2}\right)\left(x + \frac{i\sqrt{7}}{2}\right)$$

Or

$$(2x - i\sqrt{7})(2x + i\sqrt{7})$$

5) $11x^2 + 6$

$$11\left(x - \frac{i\sqrt{66}}{11}\right)\left(x + \frac{i\sqrt{66}}{11}\right)$$

Or

$$(\sqrt{11}x - i\sqrt{6})(\sqrt{11}x + i\sqrt{6})$$

6) $2v^2 + 7$

$$2\left(v - \frac{i\sqrt{14}}{2}\right)\left(v + \frac{i\sqrt{14}}{2}\right)$$

Or

$$(\sqrt{2}v - i\sqrt{7})(\sqrt{2}v + i\sqrt{7})$$

7) $4x^2 + 4x + 5$

$$4\left(x + \frac{1 - 2i}{2}\right)\left(x + \frac{1 + 2i}{2}\right)$$

Or

$$4\left(x + \frac{1}{2} - i\right)\left(x + \frac{1}{2} + i\right)$$

8) $4x^2 + 8x + 6$

$$4\left(x + \frac{2 - i\sqrt{2}}{2}\right)\left(x + \frac{2 + i\sqrt{2}}{2}\right)$$

Or

$$4\left(x + 1 - \frac{i\sqrt{2}}{2}\right)\left(x + 1 + \frac{i\sqrt{2}}{2}\right)$$

9) $9n^2 - 5n + 12$

$$9\left(n - \frac{5 + i\sqrt{407}}{18}\right)\left(n - \frac{5 - i\sqrt{407}}{18}\right)$$

Or

$$9\left(n - \frac{5}{18} - \frac{i\sqrt{407}}{18}\right)\left(n - \frac{5}{18} + \frac{i\sqrt{407}}{18}\right)$$

11) $3n^2 - 7n + 12$

$$3\left(n - \frac{7 + i\sqrt{95}}{6}\right)\left(n - \frac{7 - i\sqrt{95}}{6}\right)$$

Or

$$3\left(n - \frac{7}{6} - \frac{i\sqrt{95}}{6}\right)\left(n - \frac{7}{6} + \frac{i\sqrt{95}}{6}\right)$$

13) $4a^2 + 12a + 10$

$$4\left(a + \frac{3 - i}{2}\right)\left(a + \frac{3 + i}{2}\right)$$

Or

$$4\left(a + \frac{3}{2} - \frac{i}{2}\right)\left(a + \frac{3}{2} + \frac{i}{2}\right)$$

15) $12v^2 - 3v + 2$

$$12\left(v - \frac{3 - i\sqrt{87}}{24}\right)\left(v - \frac{3 + i\sqrt{87}}{24}\right)$$

Or

$$12\left(v - \frac{1}{8} + \frac{i\sqrt{87}}{24}\right)\left(v - \frac{1}{8} - \frac{i\sqrt{87}}{24}\right)$$

10) $8p^2 + 5p + 2$

$$8\left(p + \frac{5 - i\sqrt{39}}{16}\right)\left(p + \frac{5 + i\sqrt{39}}{16}\right)$$

Or

$$8\left(p + \frac{5}{16} - \frac{i\sqrt{39}}{16}\right)\left(p + \frac{5}{16} + \frac{i\sqrt{39}}{16}\right)$$

12) $10a^2 + 3$

$$10\left(a - \frac{i\sqrt{30}}{10}\right)\left(a + \frac{i\sqrt{30}}{10}\right)$$

Or

$$(\sqrt{10}a - i\sqrt{3})(\sqrt{10}a + i\sqrt{3})$$

14) $9n^2 - 12n + 10$

$$9\left(n - \frac{2 - i\sqrt{6}}{3}\right)\left(n - \frac{2 + i\sqrt{6}}{3}\right)$$

Or

$$9\left(n - \frac{2}{3} + \frac{i\sqrt{6}}{3}\right)\left(n - \frac{2}{3} - \frac{i\sqrt{6}}{3}\right)$$

16) $2n^2 + 6$

$$2(n - i\sqrt{3})(n + i\sqrt{3})$$

Or

$$(\sqrt{2}n - i\sqrt{6})(\sqrt{2}n + i\sqrt{6})$$

$$17) 12a^2 + 7$$

$$12 \left(a - \frac{i\sqrt{21}}{6} \right) \left(a + \frac{i\sqrt{21}}{6} \right)$$

Or

$$(2\sqrt{3}a - i\sqrt{7})(2\sqrt{3}a + i\sqrt{7})$$

$$19) 4x^2 - 7x + 4$$

$$4 \left(x - \frac{7 + i\sqrt{15}}{8} \right) \left(x - \frac{7 - i\sqrt{15}}{8} \right)$$

Or

$$4 \left(x - \frac{7}{8} - \frac{i\sqrt{15}}{8} \right) \left(x - \frac{7}{8} + \frac{i\sqrt{15}}{8} \right)$$

$$18) 9x^2 + 12x + 6$$

$$9 \left(x + \frac{2 - i\sqrt{2}}{3} \right) \left(x + \frac{2 + i\sqrt{2}}{3} \right)$$

Or

$$9 \left(x + \frac{2}{3} - \frac{i\sqrt{2}}{3} \right) \left(x + \frac{2}{3} + \frac{i\sqrt{2}}{3} \right)$$

$$20) 6r^2 - 4r + 11$$

$$6 \left(r - \frac{2 - i\sqrt{62}}{6} \right) \left(r - \frac{2 + i\sqrt{62}}{6} \right)$$

Or

$$6 \left(r - \frac{1}{3} + \frac{i\sqrt{62}}{6} \right) \left(r - \frac{1}{3} - \frac{i\sqrt{62}}{6} \right)$$